Middle Level Number Sense

Number sense is the foundation for all mathematics. Building a sense of numbers and of number relationships is essential for developing the flexibility to deal with numbers in many forms. Mental estimations and calculations, efficient strategies, and verified solutions are fundamental in the world of work.

Students will display a good sense of numbers relating to their own experiences, creating extensions of those experiences, and exhibiting a sense for size, meaning, and a variety of expressions associated with numbers.

Measurable Performances

The learner will:

- *Use proportions, percentages, and exponential numbers to accurately solve a wide variety of problems.*
- *Demonstrate proper use of order of operations when performing calculations.*
- *Estimate and calculate mentally using number properties.*
- *Recognize, relate, and utilize real number systems.*
- *Justify reasonableness of solutions.*

A Closer Look

Teachers can promote student development of number sense by using pictorial representations such as number lines, area models, and graphs. Understanding the meaning of numbers and the relationships between them requires a change in the approach to teaching. Lecture or drill methods should be replaced by student-centered methods which focus on solution strategies. For example, students should keep a math notebook where they write the definition of properties and hints, clues, shortcuts, etc. for problem solving.

The introduction of negative numbers at the middle level should be relevant to real situations. The concept of percentages and their fractional and decimal equivalents will be reinforced. Exponential numbers and scientific notation are best used in association with the technology needed to express the magnitude of numbers and scientific applications. Use of appropriate names for number properties such as commutative, associative, and distributive is helpful in communicating mathematically, but is not essential to students' development of number sense. However, applying these properties for orderly mathematical

calculation is imperative. Mental mathematics is directly dependent on these properties.

Studying the history of the development of number systems or discussing alternate number systems such as the binary system may be helpful. Such activities help students develop an understanding of number relationships and an appreciation of their importance in different cultural settings.

Sample Investigation

Students collect articles on various themes containing numerical data such as weather data, nutritional analyses, stock market data, and sports statistics. Students should explore how the numbers can be expressed in different forms (percentages, fractions, decimals, etc.) and relate numbers to common experiences. Rewrite the articles and draw a conclusion about the theme being investigated.

Middle Level Measurement

Measurement is a powerful tool used across the curriculum and in everyday life. It is an active exploration of the world. Measurement activities develop concepts and skills used to solve problems and investigate other situations. Connections to science, geometry, and number sense are made at this level. Direct and indirect measurements are used daily for various applications. The concept of indirect measurement is introduced to determine accurate measures of inaccessible objects, irregular shapes, and curved surfaces.

Students are responsible for the selection of tools, interpretation of results, and application of findings. Students develop confidence in their skill to measure and estimate.

Measurable Performances

The learner will:

- ***Use*** *concrete models for problem solving.*
- ***Select*** *and utilize appropriate units and tools of measurement for various quantities.*
- *Demonstrate accuracy in measurement to various levels of precision.*

A Closer Look

Measurement tools can be created to ensure that each student experiences the activities of measurement in the school and at home. Angle measure, capacity, weight, and mass units are used for estimating and determining the degree of precision.

Perimeter, area, and volume formulas are developed and relationships established. Changing measurements to observe and predict the effect on perimeter, area, and volume can be modeled on spreadsheets.

Scale drawings can be interpreted using proportions. Models or drawings can be constructed. Road and topographical maps are used to represent large distances and establish connections to other topics such as science and social studies.

Sample Investigation #1

Make your own measurement tool and develop your own nonstandard unit of measure. As part of a math center, include objects (e.g., string, sheets of paper, containers, etc.) and have students estimate length, area, or volume in standard and nonstandard units.

Notes to Teacher

- Create a protractor by paper folding.
- Use paper folding to create measurement tools for length.
- Make a barometer with flask and glass tubes.

Extensions

- Use grid paper to estimate area of irregular shapes.
- Estimate distances such as across a lake, a driving distance, or a long jump.

Sample Investigation #2

A unit on outdoor math could include drawing maps by pacing, creating scale drawings of objects, and finding a location given angles and distances to travel. Similar triangles could be used to estimate and then to determine the height of a tree or a flagpole. A sundial could also be created.

Middle Level Spatial Relationships/Geometric Topics

Students develop spatial sense by touching, building, comparing, drawing, measuring, and classifying geometric figures. Investigating the relationships and properties among geometric figures leads to the development and understanding of formulas and definitions, rather than memorizing them. Students create geometric models to analyze and solve problems. Geometric topics at this level should serve as a transition from the informal organization in the primary and intermediate levels, to the more formalized approach in high school.

Measurable Performances

The learner will:

- *Represent and solve actual problems using geometric models.*
- *Describe, classify, and compare geometric figures.*

  Recognize and utilize transformations.
- *Communicate with relevant vocabulary.*
- *Use basic constructions to create models.*

A Closer Look

At the middle level, informal geometry and spatial awareness are essential. Physical models help students explore and classify polygons and polyhedra, discover relationships, and determine formulas. Discovering connections between the formulas and models are made with less emphasis on memorization.

For example, the Pythagorean Theorem connects to practical applications such as diagonal distance as well as exponential notation. The basic trigonometric ratios of sine, cosine, and tangent follow as a natural extension. Cooperative group activities using problems from actual settings help students see connections. Tessellations, flips, slides, and turns have applications in art, tiling, and packaging. Exploring the history of various mathematicians leads to an appreciation of the evolution of geometric topics.

Students' investigations with manipulatives, models and activities such as paper folding help develop concepts of geometry. Students should investigate common angle measurements to be able to estimate angles found in inclines such as pitch of roofs, ski slopes, etc. A geometry lab could include such

items as compasses, straight edges, tangrams, geoboards, pattern blocks, yarn, pipe cleaners, toothpicks, straws, playdough, marshmallows, patty paper, various containers, mirrors, calculators, and computers with supporting software.

Sample Investigation #1

Have students make a scale model of a room in their home.

Notes to Teacher

- Grid paper may be helpful.
- Computer Aided Drafting programs or geometric software could be used.

Sample Investigation #2

Students explore and find tessellations from objects found in the community. Make a rubbing, if possible. Students create their own tessellating design.

Middle Level Data Analysis

In a society where vast amounts of information are being generated, it is important for students to collect, organize, analyze, interpret, and draw conclusions from the data. Statistics is the process used to accomplish this task.

As students grow, they are continually confronted with choices. Beneath each choice is the probability that their decision will be successful. A collection of consequences leads to concepts about probability which need to be developed and formalized to allow intelligent decisions.

Measurable Performances

The learner will:

- *Explore, read, and interpret data presented in a variety of formats to justify conclusions.*
- *Select and utilize informal measures of central tendency to analyze data*
- *Represent data in various forms.*
- *Apply techniques of probability to make predictions.*

A Closer Look

Middle level students have a curiosity about how they compare to their peers. Information collected about students is utilized when students are computing mean, median, mode, and range. Stem and leaf or box and whisker plots can be used to compare sets of data.

Connections can be made to other disciplines such as sports statistics in physical education and genetic probability in science. Students are involved in the whole process which includes developing the survey, collecting the data, creating and presenting the information in a variety of graphs, and drawing conclusions. This allows students to analyze better other graphs and to determine how the same information can be used to demonstrate different points of view. For example, the slope of a city's population graph over time can reflect an increasing or static population by changing the scale on the vertical axis. Computer programs quickly change the scale to demonstrate different views. The concept of random sampling is used in collecting the data from a large group and making predictions about the total group. Commercials can be analyzed to help students make informed consumer decisions.

Students' concepts about probability develop from informal processing of consequences of events occurring in their surroundings. When facing a new or uncertain situation, students rely on misconceptions about properties of chance to make a decision. By actively involving students in experimenting, making predictions, and analyzing results, students can begin to formalize the concepts regarding probability. The probability of a event is experimentally and theoretically developed. Experimental results can vary from the theoretical probability. Students should realize that the larger number of trials performed will provide an answer closer to the theoretical probability.

Sample Investigation #1

Students will decide on a question to serve as the basis for a survey, such as number of people in family, a favorite movie, etc. A method to collect data will be created; data will be collected, displayed in various formats, and conclusions will be drawn from the results.

Sample Investigation #2

The concept of what is a "fair" game will help students develop concepts of probability. A game might be rolling two dice: one person wins if the sum of the dice is 2, 3, 4, 9, 10, 11, or 12, and the other person wins if the sum is a 5, 6, 7, or 8. Students decide which person they wish to be and play the game to see who wins. After playing the game, students analyze their choices and examine the theoretical probability for each.

Extensions

- Calculators can be used to compute standard deviations of groups of data to compare groups with similar mean, median, mode, and range, but with different sets of data.

Sample Investigation #3

Using a display of data from a newspaper, determine the source of the data, what conclusions could be drawn from the data, and whether the information accompanying the display seems reasonable.

Middle Level Patterns and Functions

The study of patterns in real-world phenomena develops an appreciation of mathematics in nature. Students are exposed to patterns in many disciplines. Individual differences in interpretation of patterns should be encouraged and explored. At this level students will begin to organize and shift their attention to functions.

Measurable Performances

The learner will:

- *Represent and verify the relationship between sets of data with functions.*
- *Recognize a wide variety of patterns and effectively communicate relationships.*
- *Discover and utilize patterns to solve problems.*
- *Recognize the importance of patterns in mathematics.*

A Closer Look

The power of patterns allows middle level students to extend their thinking skills to a higher level of understanding. Connections are made to art, music, and nature in the global environment.

Students represent and relate patterns in a variety of ways which may include tessellations, Venn diagrams, or graphs. Special patterns such as the Fibonnacci numbers, Pascal's Triangle, the works of Escher, chaos theory, and the Golden ratio may be explored.

Number sense is enhanced by the use of patterns. Foundations for further development in algebra and calculus are provided by relating patterns to functions, domain, range, maximum, and minimum.

Supporting tools for the investigation of patterns include manipulatives, computer software, spreadsheets, calculators, and graphing calculators.

Sample Investigation

Explore occurrences of the Golden ratio in the students' environments.

Notes to Teacher

- Find the Golden ratio by having students measure and compare:
  a. The distance between the successive joints on their index finger.
  b. The distance between their waist and the floor compared to their total height.
- Shapes which are visually appealing will have ratios close to the Golden ratio. Consider occurrences of the Golden rectangle by comparing the ratio of length to width. Use objects in the classroom such as a poster, a desk top, or the cover of a book. Extend to examples outside the classroom making sure that students verify the relationship

Extensions

- An introduction to fractals and Sierpinski's Triangle.
- An introduction to networks and paths.

Middle Level Algebraic Topics

Algebra is a method of communication which involves the use of symbols to represent physical quantities or numbers. At the middle level, algebraic topics begin the transition from concrete to abstract. Expressions and equations are used to represent the relationship between quantities.

Measurable Performances

The learner will:

- *Develop appropriate mathematical models to represent actual situations.*
- *Determine solutions to linear equations and inequalities.*
- *Evaluate variable expressions by substituting values and representing results appropriately.*
- *Explore and analyze relationships between quantities.*

A Closer Look

Middle level students' introduction to algebraic topics must be concrete and manipulative-based. A mathematics lab could include: algebra tiles, rods, balance scale, computers with supporting software, spreadsheets, scientific calculators, and graphing calculators. Students will develop concepts which lead to formal strategies for solving equations in future courses. It is important that students experience success and not be pushed into abstract solving of equations before they are ready. Algebra is used as a problem-solving strategy.

Algebra should not be limited to solving equations or plotting simple linear equations but should demonstrate the relationship between quantities. Students will solve problems that have multiple solutions. Connections between science and mathematics should be demonstrated by using scientific formulas with direct and inverse variation.

Sample Investigation #1

Collect samples of hourly wages for several occupations, and compare how much is earned over a period of time.

Sample Investigation #2

Compare height against time for a person parachuting out of an airplane to show how height changes in relation to time. Other comparisons can be made between height versus shoe size, calories burned versus physical activity, and speed versus stopping distance.

Notes to Teacher

- Students could examine differences in hourly wages with respect to gender, occupations, location, etc. These differences will be evident in the slope of the lines.

Extensions

- Students may be exposed to graphing and solving systems of equations. Students may explore equations using a graphics calculator.